Natural Mathematics

Core Axioms and Derived Structure

Core Principle: Operator, Not Arithmetic

Natural Maths reformulates number as orientation and operation.
Structure arises from the simplest geometric constraints.
Counting emerges; geometry is fundamental.

1. Axioms

Four axioms define the necessary "number geometry" of the Natural Number Field.

Axiom 1 — Duality Identity

$$x^2 = -x$$

This symmetry identity defines the minimal nontrivial real structure.

Consequences:

- Complex rotation collapses:

$$\sqrt{-1} = 1$$

(orientation, not magnitude)

- Only two orientations exist:

$$\sigma \in \{-1, +1\}$$

Axiom 2 — Orientation Principle

Every state carries an intrinsic sign-orientation:

$$\sigma_n \in \{-1, +1\}$$

This is a primitive geometric property (analogous to phase or spin).

Axiom 3 — Canonical Iteration Rule

There is one and only one quadratic dynamic compatible with the 2 previous axioms:

$$x_{n+1} = \sigma_n x_n^2 + c$$

This is the unique (fundamental) quadratic map of natural mathematics.

Axiom 4 — Orientation Persistence

In the canonical system:

$$\sigma_{n+1} = \sigma_n$$

Orientation persists unless externally perturbed.

2. Definitions

Definition — 2: The Cut Operator

2 is not a structural number.
It is the operator that imposes perfect symmetry and flips orientation.
It generates the duality of the system.

Thus 2 is excluded from the Natural Primes.

Definition — Natural Primes

These are the structural excitations not produced by the Cut Operator:

$$\mathbb{P}_{\mathsf{NM}} = \{3,5,7,11,\ldots\}$$

All gaps are even.

Definition — Curvature-Sensitivity Parameter κ

κ is a **probe**. It introduces controlled perturbations to Axiom 4:

$$\sigma_{n+1} = -\sigma_n \quad \text{if a κ-dependent condition holds}$$

κ allows exploration of curvature sensitivity and stability spectra.

It is for *diagnostic analysis only*.

3. The Natural-Maths Mandelbrot Set

$$\mathscr{M}_{\mathsf{NM}} = \left\{ c \in \mathbb{R} : x_{n+1} = \sigma x_n^2 + c,\ \sigma = \pm 1,\ |x_n| \nrightarrow \infty \right\}$$

This object is uniquely determined by the axioms.

- x-axis: parameter c
- y-axis: initial orientation bias (via b → σ_0)

4. Theorem — Uniqueness of the NM Mandelbrot Set

Because:

- Complex rotation is forbidden
- Only two orientations exist
- The quadratic map is uniquely forced
- Orientation is persistent

there is only **one** Mandelbrot set in Natural Maths and no alternative formulation.

5. Geometric Validation (Empirical, Not Axiomatic)

κ-perturbations reveal:

- κ = 2 → maximal symmetry (signature of the Cut Operator)
- κ = 3 → first structural excitation (first Natural Prime)
- κ → ∞ → Feigenbaum-like cascade
- κ < 0 → mirrored duality structure

These behaviours validate the axiom system but do not define it.

6. (Optional) Physical Interpretation Layer

Natural Maths permits—but does not require—physical mappings:

- κ as curvature sensitivity
- $\kappa \approx 0.624$ linked to GUE-type spectra
- possible connections to stability, gravity, or cosmological structure